Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

1 May 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2224 – 7101

14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Claire rolls a six-sided die 16 times.

The scores obtained are shown in the following frequency table.

Score	Frequency
1	p
2	q
3	4
4	2
5	0
6	3

It is given that the mean score is 3.

(a) Find the value of p and the value of q. [5]

Each of Claire's scores is multiplied by 10 in order to determine the final score for a game she is playing.

(b) Write down the mean final score. [1]

2. [Maximum mark: 5]

It is given that $\log_{10} a = \frac{1}{3}$, where $a > 0$.

Find the value of

(a) $\log_{10}\left(\frac{1}{a}\right)$; [2]

(b) $\log_{1000} a$. [3]

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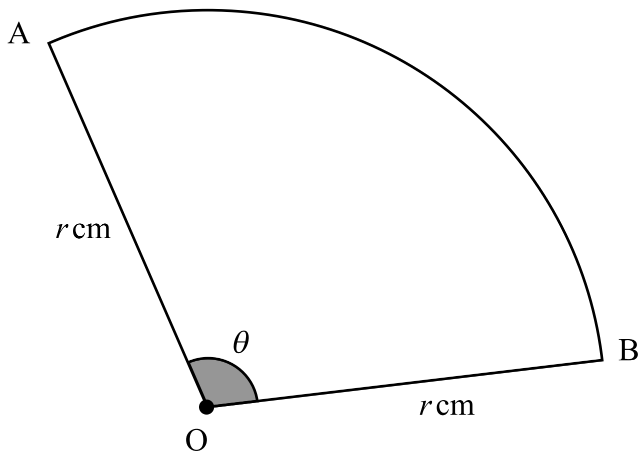
3. [Maximum mark: 8]

Points A and B lie on the circumference of a circle of radius $r\,\mathrm{cm}$ with centre at O.

The sector OAB is shown on the following diagram. The angle $\mathrm{A\hat{O}B}$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is $10\,\mathrm{cm}$ and the area of the sector is $6.25\,\mathrm{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

(b) Hence, or otherwise, find the value of r and the value of θ. [4]

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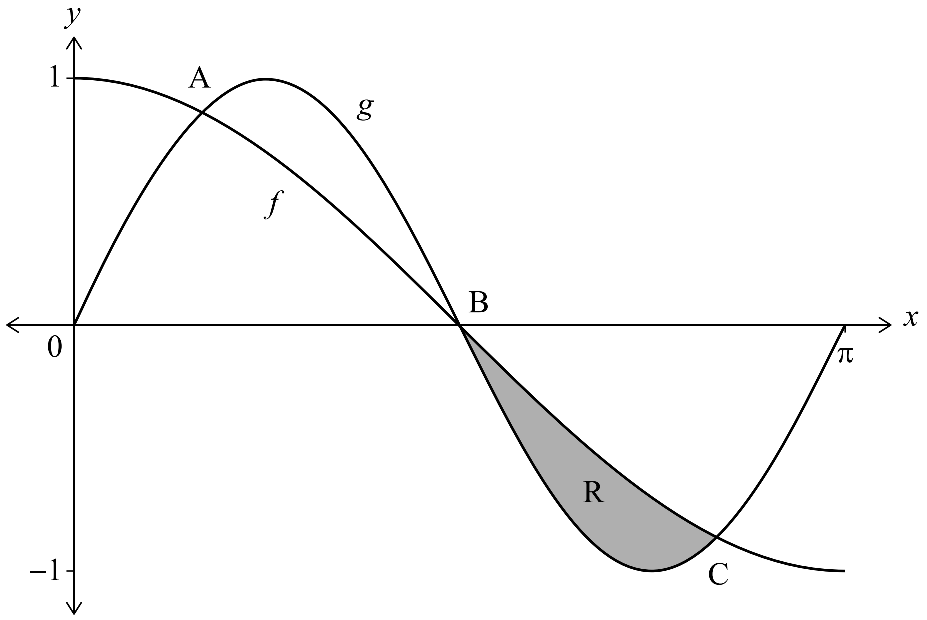
4. [Maximum mark: 7]

Consider the functions $f(x) = \cos x$ and $g(x) = \sin 2x$, where $0 \le x \le \pi$.

The graph of f intersects the graph of g at the point A, the point B $\left(\frac{\pi}{2}, 0\right)$ and the point C as shown on the following diagram.



(a) Find the x-coordinate of point A and the x-coordinate of point C. [3]

The shaded region R is enclosed by the graph of f and the graph of g between the points B and C.

(b) Find the area of R. [4]

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5. [Maximum mark: 5]

Consider a geometric sequence with first term 1 and common ratio 10.

S_n is the sum of the first n terms of the sequence.

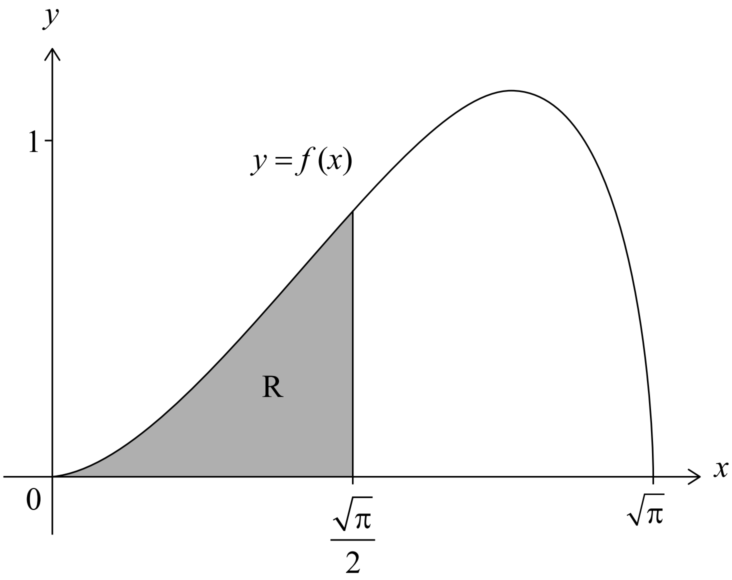
(a) Find an expression for S_n in the form $\frac{a^n - 1}{b}$, where $a, b \in \mathbb{Z}^+$. [1]

(b) Hence, show that $S_1 + S_2 + S_3 + \ldots + S_n = \frac{10\left(10^n - 1\right) - 9n}{81}$. [4]

6. [Maximum mark: 6]

The function f is defined as $f(x) = \sqrt{x\sin(x^2)}$, where $0 \le x \le \sqrt{\pi}$.

Consider the shaded region R enclosed by the graph of f, the x-axis and the line $x = \frac{\sqrt{\pi}}{2}$, as shown in the following diagram.



The shaded region R is rotated by 2π radians about the x-axis to form a solid.

Show that the volume of the solid is $\frac{\pi(2-\sqrt{2})}{4}$.

7. [Maximum mark: 7]

Using mathematical induction and the definition ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$, prove that $\sum_{r=1}^{n} {}^{r}C_{1} = {}^{n+1}C_{2}$ for all $n \in \mathbb{Z}^{+}$.

8. [Maximum mark: 7]

(a) Find the first two non-zero terms in the Maclaurin series of

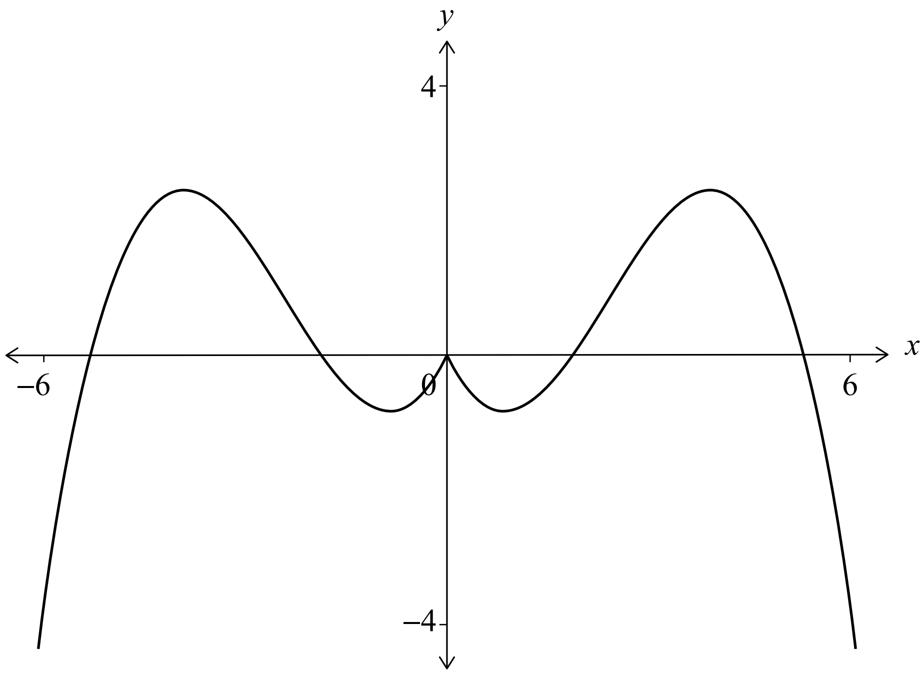
(i) $\sin\left(x^2\right)$;

(ii) $\sin^2\left(x^2\right)$. [5]

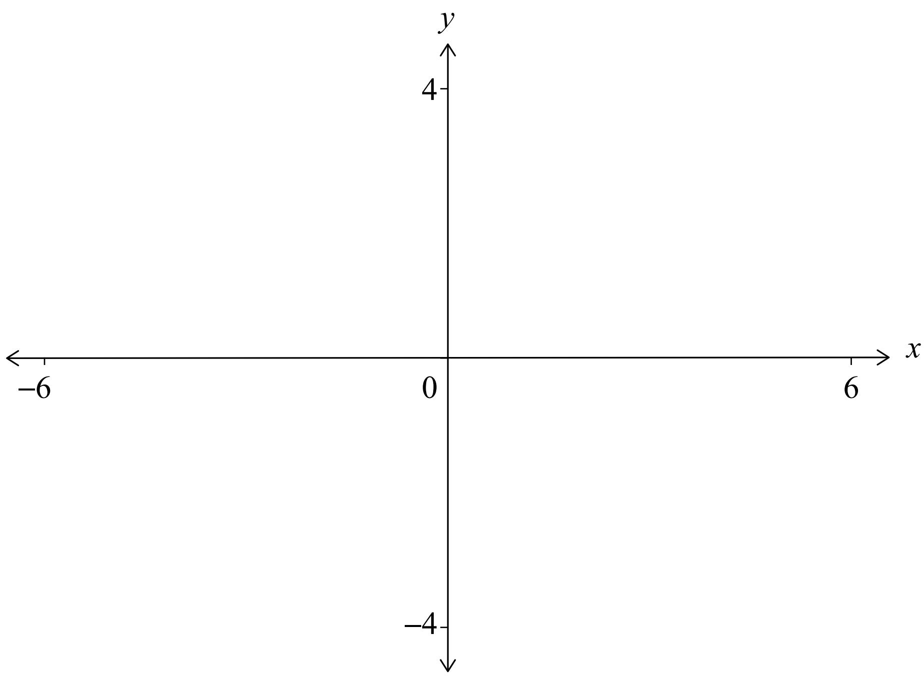
(b) Hence, or otherwise, find the first two non-zero terms in the Maclaurin series of $4x\sin\left(x^2\right)\cos\left(x^2\right)$. [2]

9. [Maximum mark: 6]

The graph of $y = f(|x|)$ for $-6 \le x \le 6$ is shown in the following diagram.



(a) On the following axes, sketch the graph of $y = |f(|x|)|$ for $-6 \le x \le 6$. [2]



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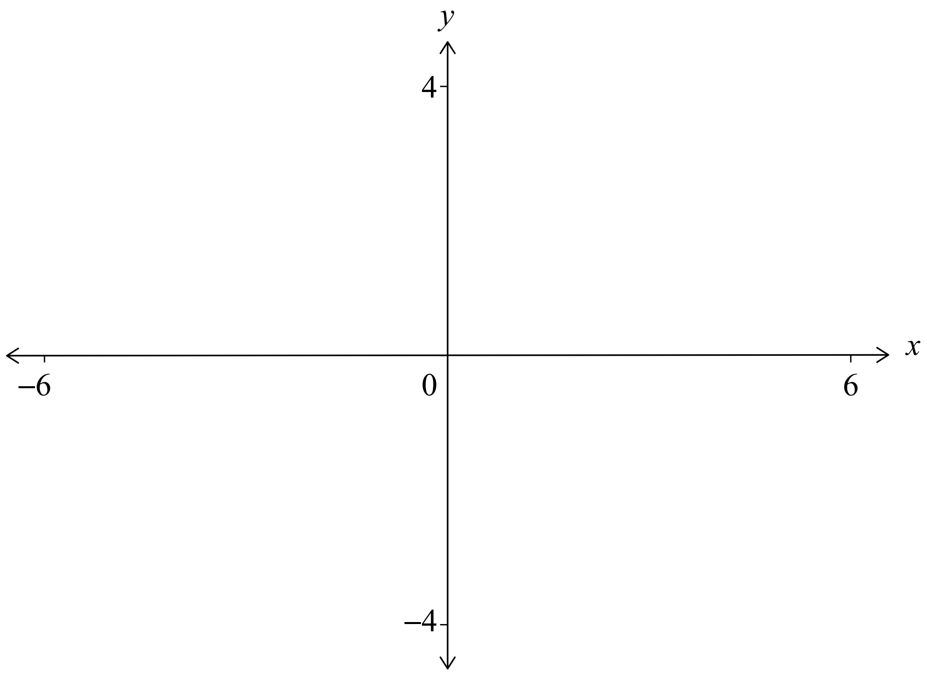
(Question 9 continued)

It is given that f is an odd function.

(b) On the following axes, sketch the graph of $y = f(x)$ for $-6 \le x \le 6$. [2]



It is also given that $\int_0^4 f(|x|)\,\mathrm{d}x = 1.6$.

(c) Write down the value of

(i) $\int_{-4}^{0} f(x)\,\mathrm{d}x$;

(ii) $\int_{-4}^{4} \left(f(|x|) + f(x) \right) \mathrm{d}x$. [2]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f. [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

(d) Find the value of b and the value of c. [3]

(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the product of the solutions of the equation $f(x) = g(x)$. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

Consider the polynomial $P(x) = 3x^3 + 5x^2 + x - 1$.

(a) Show that $(x + 1)$ is a factor of $P(x)$. [2]

(b) Hence, express $P(x)$ as a product of three linear factors. [3]

Now consider the polynomial $Q(x) = (x + 1)(2x + 1)$.

(c) Express $\dfrac{1}{Q(x)}$ in the form $\dfrac{A}{x+1} + \dfrac{B}{2x+1}$, where $A, B \in \mathbb{Z}$. [3]

(d) Hence, or otherwise, show that $\dfrac{1}{(x+1)Q(x)} = \dfrac{4}{2x+1} - \dfrac{2}{x+1} - \dfrac{1}{(x+1)^2}$. [2]

(e) Hence, find $\displaystyle\int \frac{1}{(x+1)^2(2x+1)}\,\mathrm{d}x$. [4]

Consider the function defined by $f(x) = \dfrac{P(x)}{(x+1)Q(x)}$, where $x \neq -1$, $x \neq -\dfrac{1}{2}$.

(f) Find

(i) $\lim\limits_{x \to -1} f(x)$;

(ii) $\lim\limits_{x \to \infty} f(x)$. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 20]

Consider $\phi = (a + b\mathrm{i})^3$, where a, $b \in \mathbb{R}$.

(a) In terms of a and b, find

(i) the real part of ϕ;

(ii) the imaginary part of ϕ. [3]

(b) Hence, or otherwise, show that $\left(1+\sqrt{3}\,\mathrm{i}\right)^3 = -8$. [2]

The roots of the equation $z^3 = -8$ are u, v and w, where $u = 1+\sqrt{3}\,\mathrm{i}$ and $v \in \mathbb{R}$.

(c) Write down v and w, giving your answers in Cartesian form. [2]

On an Argand diagram, u, v and w are represented by the points U, V and W respectively.

(d) Find the area of the triangle UVW. [3]

Each of the points U, V and W is rotated counter-clockwise (anticlockwise) about 0 through an angle of $\frac{\pi}{4}$ to form three new points U', V' and W'. These points represent the complex numbers u', v' and w' respectively.

(e) Find u', v' and w', giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi < \theta \le \pi$. [4]

(f) Given that u', v' and w' are the solutions of $z^3 = c + d\mathrm{i}$, where c, $d \in \mathbb{R}$, find the value of c and the value of d. [3]

It is given that u, v, w, u', v' and w' are all solutions of $z^n = \alpha$ for some $\alpha \in \mathbb{C}$, where $n \in \mathbb{N}$.

(g) Find the smallest positive value of n. [3]

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will not be marked.

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